On the Robust and Efficient Computation of the Kalman Gain for Multichannel Adaptive Filtering with Application to Acoustic Echo Cancellation

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***Abstract*—In this paper we discuss efficient approaches for a robust computation of the spatio-temporal Kalman gain in the context of adaptive multichannel systems. We apply the given approaches to the multichannel acoustic echo cancellation problem as representative for the class of system identification problems. Moreover, we provide a complexity and performance analysis of our approaches and compare the results to the conventional spatio-temporal Kalman gain computation methods.**

I. Introduction

A fundamental building block of adaptive filters for system identification (in the least squares sense) is the computation of a numerically stable Kalman gain. The computation of the Kalman gain requires, in principle, an inversion of a correlation matrix. In a straightforward realization, this inversion in single-channel adaptive filtering problems has a complexity of $O(L^3)$, where L is the filter length, which is typically equal to the temporal processing block-length in block-by-block adaptive processing. However, using the Woodbury-identity (the so-called matrix inversion lemma) reduces the inversion complexity to $O(L^2)$. Moreover, when a transform domain is known a priori where the correlation matrix is diagonal, a matrix inversion in such a domain can be done in linear time. But here, the complexity of the transformation into this domain has to be taken into account. Advanced adaptive filtering algorithms, such as Fast-RLS reach an overall complexity of $O(L)$. This can be done by exploiting the FIR structure of the temporal signal model.

Advanced audio reproduction and capturing systems, e.g., systems based on wave field synthesis and analysis (WFS/WFA) [1], require closely spaced arrays of a large number of loudspeakers and/or microphones to achieve a high spatial resolution. The number of loudspeakers can lie up to several hundreds emitting, in general, highly correlated signals. It has been shown that treating adaptive filtering problems for such broadband massive multichannel systems in spatio-temporal transform domains is a powerful approach [2], [3]. Spatio-temporal transform-domain techniques for multichannel adaptive systems aim at decoupling the signal correlation matrix both in the temporal and in the spatial dimensions [2], [3]. Therefore, the decoupling process consists of two steps: (1) temporal decoupling based on the discrete Fourier transform (DFT) and (2) a spatial decoupling using a unitary transform. When the spatial decoupling matrix is a priori known, the computation complexity of the Kalman gain is dominated by the transformation complexity. Since the spatial transformation can be done by a matrix-vector multiplication [4], the complexity will be in $O(L \cdot P^2)$, where P is the number of the adaptive system input channels (loudspeakers).

When the decorrelating spatial transformation domain is not a priori known the complexity of a straightforward computation will be in $O(L{\cdot}P^3)$. In [5] an approach is proposed for an efficient Kalman gain computation (in $O(L \cdot P^2)$). This approach is based on the inversion lemma. But since the inversion lemma holds only for matrices with full rank and the input signals of the adaptive systems are usually spatially correlated, the autocorrelation must be regularized. Unfortunately, the estimation of the regularization parameters poses a challenge for real-time applications.

In this paper we discuss two efficient methods for dealing with computational complexity and the numerical stability of the Kalman gain. These two possibilities exploit the additive update process of the hermitian correlation matrix.

- The first method we investigate is to use the theory of rank-one modification of the eigenvalue problem [6]. For an efficient computation we exploit the approximation given in [7] based on the interleaf theorem. Subsequently, we apply a dynamic regularization for a stable inversion of the eigenvalues.
- Another discussed method in this paper is using the generalized inversion of modified matrices as proposed in [8] for our purposes.

Both approaches are then applied in the context of massive multichannel acoustic echo cancellation (AEC) as a representative for the class of system identification algorithms.

Fig. 1 shows a block diagram of multichannel AEC with P reproduction channels and Q microphone channels in the receiving room ('near end'). The signals of the P reproduction channels originate from speech- or audio sources in a transmission room ('far end'). To cancel the echoes arising due to the reflections in the near end, an adaptation algorithm estimates the matrix $\mathbf{H}$ of acoustic impulse responses from the loudspeakers to the microphones. The estimated coefficients matrix $\widehat{\mathbf{H}}$ denotes the $PL \times Q$ MIMO coefficient matrix composed by $P \cdot Q$ subfilters, $\hat{\mathbf{h}}_{pq} = [\hat{h}_{pq,0}, \hat{h}_{pq,1}, \cdots, \hat{h}_{pq,L-1}]^T$. The reproduction signals $x_p(n)$, where n denotes a time instant, are filtered with the estimated coefficients $\widehat{\mathbf{H}}(n)$, the resulting signals are subtracted from the near-end microphone signals $y_q(n)$. If the estimated filter coefficients $\widehat{\mathbf{H}}$ are equal to the true transfer paths $\mathbf{H}$, all disturbing echoes will be removed from the microphone signals.

II. Transform domain adaptive filtering

Single channel adaptive filtering problems are often considered in a convenient transform domain, namely, the discrete Fourier transform domain. The motivation to choose this transform domain originates from the fact that the Fourier basis functions are eigenfunctions of linear time invariant (LTI) systems. Therefore, the filtering of the input signal with the filter coefficients of the adaptive filter can be performed efficiently as fast convolution in the frequency domain by exploiting the efficiency of the fast Fourier transform (FFT).

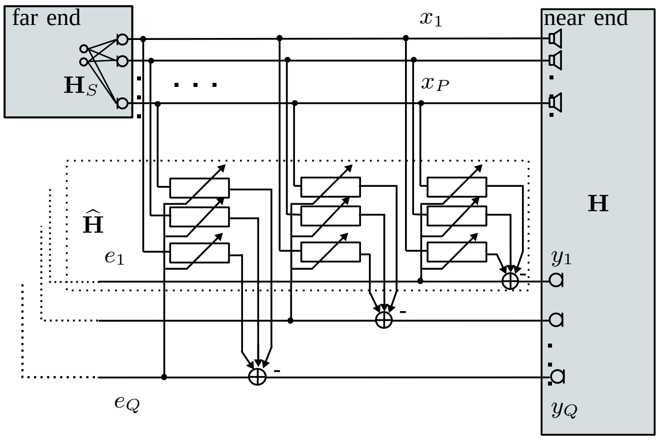

Fig. 1. Block diagram of multichannel acoustic echo cancellation as a prominent example for adaptive MIMO filtering.

On the other hand, under the assumption of stationary signals, the covariance matrix of the input signals is approximately diagonalized by the discrete Fourier transformation (DFT). Hence, this domain allows a computationally efficient inversion of the covariance matrix by considering only the elements on the main diagonal. In this way, it is possible to efficiently take all temporal correlations into account. For multiple input/multiple output (MIMO) systems analogous considerations can be made. The temporal component can be treated similarly to the single channel case. Hence, the intra-channel autocorrelation matrix can be diagonalized by the DFT. Additionally, the spatial component should be taken into account, thus the channels have to be combined into suitably chosen modes by a domain transformation.

The strategies for the spatio-temporal transform-domain adaptive filtering can be classified into two groups depending on the a-priori knowledge on the MIMO system. These two groups are briefly reviewed next.

A. Spatio-temporal transform-domain adaptive filtering with respect to the system

A-priori knowledge about the system can be obtained from the physics underlying the system under study. An example of exploiting such a-priori knowledge about electro-acoustic systems is the approach of wave domain adaptive filtering (WDAF)[2], [4].

Acoustic wave propagation from a source to a point in the space can be understood as multidimensional system which is characterized by the inhomogeneous wave equation with respect to the present boundary conditions. A major problem of this approach is that the eigenfunctions are analytically only available for rather simple geometries (e.g. sphere, box) and boundary conditions. The basic concept of WDAF is to use analytic eigenfunctions that only approximately decouple the system and perform the adaptive filtering in the eigenspace by transforming the input and output signals. The coefficients of the transformation filters can be derived from sampling the eigenfunctions. The free-field solutions of the wave equation in suitable coordinate systems have shown good performance.

B. Spatio-temporal transform-domain adaptive filtering with respect to signal statistics

In the general case of acoustic echo cancellation scenarios a-priori system knowledge is absent and the system is poorly excited. The excitation can be considered as poor when the eigenvalue spread or the ratio of maximum to minimum eigenvalue of the data autocorrelation matrix is large [9]. This occurs, e.g., when the number of the rendered uncorrelated sources is smaller than the number of the loudspeakers. It has been shown that cross-correlations between the loudspeaker signals let the adaptive filter converge to a solution that depends on the characteristics of the loudspeaker signals. Any movement of the sound source in the transmission room results in a breakdown of the echo cancellation performance and requires a new adaptation of the cancellation filters[10]. Therefore, a preprocessing stage to decorrelate the transmitted signals for a unique identifiability of the echo paths is required to ensure robustness to sound source movements. Preliminary experiments of the authors have shown that massive multichannel reproduction systems, e.g., WFS, are very sensitive to most of the known preprocessing techniques, especially at low frequencies. Therefore, it is desirable to avoid any manipulations of the loudspeaker signals as far as possible in terms of maintaining the desired auditory event. Hence, regularization is required, e.g., dynamical noise injection as suggested in [5] is a powerful strategy of regularization for multichannel system identification problems. Unfortunately, the estimation of the regularization parameters for massive multichannel system poses a challenge.

Therefore, we recently introduced approaches for the adaptive filtering in a spatial transform-domain depending on the signal statistic [3], [11], [12]. These techniques become related to the subspace adaptive filtering when the transform domain is constrained to be unitary. However, our approaches differ from conventional subspace adaptive filtering in the subspace tracking strategy. E.g., subspace-tracking in [13] is based on classical bi-iteration technique for slowly varying signal subspaces. Another efficient approach for the subspace-tracking is the deflation-based projection approximation subspace tracking (PASTd) algorithm [14]. In our case an immediate update of the subspace basis is required when changes in the source-domain occur to overcome the non-uniqueness problem stated above. Otherwise, the actual estimation process will be additionally delayed by the estimation process of the signal subspace and the overall performance of the echo canceler could be rated as unacceptable by the user.

III. BLOCK-BASED TRANSFORM DOMAIN ADAPTIVE FILTERING

For a practical implementation block-based algorithms are favorable. A block formulation is derived by combining L consecutive samples into blocks [3], formulating the error signal in terms of blocks and minimizing the error.

$$\mathbf{x}_p(n) = [x_p(n), x_p(n-1), \cdots, x_p(n-L+1)], \quad (1)$$

$$\mathbf{x}(n) = [\mathbf{x}_1(n), \mathbf{x}_2(n), \cdots, \mathbf{x}_p(n)]. \quad (2)$$

In this formulation the MIMO system is decomposed into a sum of MISO systems and the convolution of the input signals with each MISO system is represented by combining the input signals into a matrix with block Toeplitz structure and multiplying this input matrix with the MISO filter vector. A Toeplitz matrix can be transformed into a circulant matrix by doubling its size and a circulant matrix can be diagonalized using the $2L \times 2L$ DFT matrix $\mathbf{F}_{2L}$ with elements $e^{-j2\pi\nu n/(2L)}(\nu, n = 0, \ldots, 2L-1)$. This results in an overlap save formulation of the convolution by incorporating window functions.

The time-domain block error signal $\mathbf{e}(m)$ for a block length of L samples is defined as

$$\mathbf{e}(m) = [e(mL), e(mL+1), \cdots, e(mL+L-1)]^T, \quad (3)$$

where m denotes the block index. The microphone signal $\mathbf{y}(m)$ is defined in a similar fashion as $\mathbf{e}(m)$. In order to derive an algorithm that requires only DFTs of size $2L$, the error and microphone signals are zero padded before transformation into the frequency domain[1]

$$\underline{\mathbf{e}}(m) = \mathbf{F}_{2L}\left[\mathbf{0}_{1\times L}, \mathbf{e}^T(m)\right]^T, \quad (4)$$

and similarly for the microphone signal. The loudspeaker signals in the frequency domain are given as

$$\underline{\mathbf{X}}_p(m) = \operatorname{diag}\left\{\mathbf{F}_{2L}[x_p(mL-L), \cdots, x_p(mL+L-1)]^T\right\}, \quad (5)$$

$$\underline{\mathbf{X}}(m) = [\underline{\mathbf{X}}_1(m), \ldots, \underline{\mathbf{X}}_P(m)]. \quad (6)$$

The generic frequency-domain adaptive filtering algorithm (FDAF) for MISO systems can then be summarized as follows[5]

$$\underline{\mathbf{S}}_{xx}(m) = \lambda\underline{\mathbf{S}}_{xx}(m-1) + (1-\lambda)\underline{\mathbf{X}}^H\mathbf{G}_1\underline{\mathbf{X}}(m), \quad (7)$$

the Kalman gain reads

$$\underline{\mathbf{K}}(m) = (1-\lambda)\underline{\mathbf{S}}_{xx}^{-1}(m)\underline{\mathbf{X}}(m), \quad (8)$$

$$\underline{\mathbf{e}}'(m) = \underline{\mathbf{y}}'(m) - \mathbf{G}_2\underline{\mathbf{X}}(m)\underline{\widehat{\mathbf{h}}}'(m-1), \quad (9)$$

$$\underline{\widehat{\mathbf{h}}}'(m) = \underline{\widehat{\mathbf{h}}}'(m-1) + \mathbf{G}_3\underline{\mathbf{K}}(m)\underline{\mathbf{e}}'(m), \quad (10)$$

where λ denotes the forgetting factor and $\underline{\widehat{\mathbf{h}}}'(m)$ the zero padded vector of estimated filter coefficients which is defined as

$$\underline{\widehat{\mathbf{h}}}'(m) = \mathbf{G}^{10}_{2LP\times LP}\underline{\widehat{\mathbf{h}}}, \quad (11)$$

where $\mathbf{G}^{10}_{2LP\times LP}$ denotes a window matrix that performs the zero padding. It is defined as follows

$$\mathbf{G}^{10}_{2LP\times LP} = \operatorname{Bdiag}\left\{\mathbf{G}^{10}_{2L\times L}, \ldots, \mathbf{G}^{10}_{2L\times L}\right\}, \quad (12)$$

$$\mathbf{G}^{10}_{2L\times L} = \mathbf{F}_{2L}\left[\mathbf{I}_{L\times L}, \mathbf{0}_{L\times L}\right]^T\mathbf{F}_L^{-1}, \quad (13)$$

where $\mathbf{I}$ is the unity matrix. In the FDAF algorithm the finite block length is explicitly accounted for by the constraint matrices $\mathbf{G}_1$, $\mathbf{G}_2$ and $\mathbf{G}_3$. These are defined as

$$\mathbf{G}_1 = \mathbf{G}_2 = \mathbf{F}_{2L}\operatorname{Bdiag}\left\{\mathbf{0}_{L\times L}, \mathbf{I}_{L\times L}\right\}\mathbf{F}_{2L}^{-1}, \quad (14)$$

$$\mathbf{G}_3 = \operatorname{Bdiag}\left\{\mathbf{G}^{10}_{2L\times 2L}, \ldots, \mathbf{G}^{10}_{2L\times 2L}\right\}, \quad (15)$$

$$\mathbf{G}^{10}_{2L\times 2L} = \mathbf{F}_{2L}\operatorname{Bdiag}\left\{\mathbf{I}_{L\times L}, \mathbf{0}_{L\times L}\right\}\mathbf{F}_{2L}^{-1}. \quad (16)$$

The idea of the multichannel transform-domain adaptive filtering is

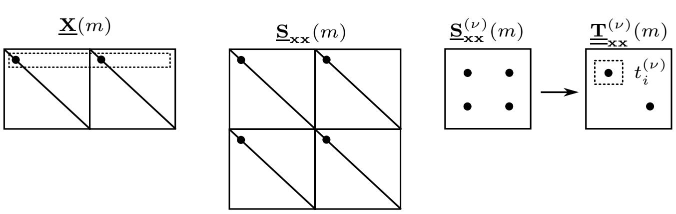

Fig. 2. Illustration of the covariance matrix and its representations for $P = 2$. The dotted box in the illustration of $\underline{\mathbf{X}}(m)$ represents $\underline{\mathbf{X}}^{(\nu)}(m)$ for (ν) a specific frequency bin.

to go one step further and diagonalize the block diagonal Fourier transformed covariance matrix, see Fig. 2. This can be done by computing the eigenvalue decomposition of the covariance matrix. The spatio-temporal transformed autocorrelation matrix in a frequency bin ν reads

$$\underline{\underline{\mathbf{T}}}^{(\nu)}_{\mathbf{xx}}(m) := \underline{\mathbf{U}}^{(\nu)}(m)\underline{\mathbf{S}}^{(\nu)}_{\mathbf{xx}}(m)\underline{\mathbf{U}}^{(\nu)H}(m),$$

hereby, $\underline{\mathbf{U}}^{(\nu)}(m)$ is the unitary transformation matrix containing the eigenvectors of the autocorrelation matrix.

Performing the adaptive filtering in the spatio-temporal transform domain offers the ability to regularize the inversion spatially and temporally frequency-bin selective. Hence, the powerful regularization strategy proposed in [5] can be generalized to the spatio-temporal domain. A regularized autocorrelation matrix in frequency bin (ν) reads

$$\underline{\underline{\tilde{\mathbf{T}}}}^{(\nu)}_{\mathbf{xx}}(m) = \underline{\underline{\mathbf{T}}}^{(\nu)}_{\mathbf{xx}}(m) + \operatorname{diag}\{\delta_i^{(\nu)}\}, \quad (17)$$

$$\delta_i^{(\nu)} = \delta_{\max}\cdot[e^{-t_1^{(\nu)}/t_0}, \cdots, e^{-t_P^{(\nu)}/t_0}]^T, \quad (18)$$

$i \in \{1, \ldots, P\}$ denotes a spatial mode, t_0 and δ_{max} are scalar parameters that should be chosen according to the (estimated) disturbing noise level in the desired signal $y(n)$.

Since the input signals are spatially non-stationary, the signal subspace will not be constant in general and therefore, the transform domain should be updated in each block [3]. To illustrate this circumstance, suppose the identification algorithm has converged in the time block $(m-1)$ to $\underline{\underline{\widehat{\mathbf{h}}}}'(m-1)$. In terms of updating this solution in the transform-domain of the block m, the filter should be transformed into the updated transform domain. This can be done as follows [3]

$$\underline{\underline{\widehat{\mathbf{h}}}}(m-1) = \mathbf{G}_{\mathbf{U}}\underline{\underline{\widehat{\mathbf{h}}}}'(m-1), \quad (19)$$

$$\mathbf{G}_{\mathbf{U}} := \underline{\mathbf{U}}^{(\nu)H}(m)\underline{\mathbf{U}}^{(\nu)}(m-1). \quad (20)$$

IV. Iterative Eigenvalue Decomposition

The complexity of computing the Kalman gain in the spatio-temporal transform domain can be reduced by exploiting the nature of the autocorrelation estimation in the transform domain

$$\underline{\underline{\mathbf{T}}}'^{(\nu)}_{\mathbf{xx}}(m) = \lambda\underline{\underline{\mathbf{T}}}^{(\nu)}_{\mathbf{xx}}(m-1) + (1-\lambda)\underline{\underline{\mathbf{X}}}'^{(\nu)H}(m)\underline{\underline{\mathbf{X}}}'^{(\nu)}(m), \quad (21)$$

$$\underline{\underline{\mathbf{X}}}'^{(\nu)}(m) := \underline{\mathbf{X}}^{(\nu)}(m)\underline{\mathbf{U}}^{(\nu)}(m-1). \quad (22)$$

The estimation is based on a rank-one modification of a diagonal matrix. Usually, the problem of determining the eigenvalues of such modified matrices is initially deflated in terms of reducing the complexity of the problem. Interesting cases are, e.g., if the modifications do not influence the eigenspace of the estimated autocorrelation matrix or if only few eigenvectors are modified, a list of the possible deflation cases can be found in [7].

Additionally, the dimension of the system to be identified shrinks from $PL\times Q$ to $kL\times Q$, k is the number of the nonzero eigenvalues. This provides a potential complexity improvement in massive multichannel systems, where the number of sources is often much smaller than the number of loudspeakers. E.g., in a conventional hands-free teleconferencing scenario the number of speakers in a far-end room is less than the number of loudspeakers which may lie up to several hundreds. It has been shown in [6] that the eigenvalues of $\underline{\underline{\mathbf{T}}}'^{(\nu)}_{\mathbf{xx}}(m)$ are the zeros of the function

$$w(t) := 1 + (1-\lambda)\cdot\sum_{i=1}^{P}\frac{\underline{\underline{x}}'^2_i}{\lambda t_i^{(\nu)}(m-1) - t}, \quad (23)$$

$$[\underline{\underline{x}}'_1 \cdots \underline{\underline{x}}'_i \cdots \underline{\underline{x}}'_k] := \underline{\mathbf{X}}^{(\nu)}(m)\underline{\mathbf{U}}^{(\nu)}(m-1),$$

[1] In this paper we use underlined symbols for frequency-domain quantities and double-underlined ones for spatially transformed quantities.

$$w(t) = 0 \text{ for } t \in \Big\{ t_i^{(\nu)}(m) \mid t_i^{(\nu)}(m) \text{ is an eigenvalue of the modified matrix } \underline{\underline{\mathbf{T}}}'^{(\nu)}_{\mathbf{xx}}(m) \Big\}.$$

The zeros of $w(t)$ can be found iteratively. However, the convergence of the search process is quadratic and good initial estimates can be obtained due to the interleaf theorem [7].

Once the eigenvalues are computed, the eigenvectors of the modified spatio-temporal transformed autocorrelation matrix $\mathbf{G}_{\mathbf{U}i}$ of $\underline{\underline{\mathbf{T}}}'^{(\nu)}_{\mathbf{xx}}(m)$ can be explicitly computed by

$$\mathbf{G}_{\mathbf{U}_i} = \frac{\underline{\underline{\mathbf{X}}}'^{(\nu)}(m)\underline{\underline{\mathbf{T}}}'^{(\nu)^{-1}}_{\mathbf{xx}_i}(m)}{\left\| \underline{\underline{\mathbf{X}}}'^{(\nu)}(m)\underline{\underline{\mathbf{T}}}'^{(\nu)^{-1}}_{\mathbf{xx}_i}(m) \right\|}, \tag{24}$$

$$\underline{\underline{\mathbf{T}}}'^{(\nu)}_{\mathbf{xx}_i}(m) := \underline{\underline{\mathbf{T}}}^{(\nu)}_{\mathbf{xx}_i}(m-1) - t_i^{(\nu)}(m) \cdot \mathbf{I}_{k\times k}. \tag{25}$$

Please note, that the matrix with the eigenvectors of the modified autocorrelation matrix is identical to the update matrix in Eq. (20) because

$$\underline{\mathbf{U}}^{(\nu)}(m) = \underline{\mathbf{U}}^{(\nu)}(m-1)\mathbf{G}^H_{\mathbf{U}}. \tag{26}$$

V. Pseudo-inversion lemma

The inversion lemma reduces the complexity of the inversion of a $P \times P$ autocorrelation matrix to $O(P^2)$. The requirement for applying the inversion lemma on a matrix is the regularity of that matrix. We propose generalizing the inversion-lemma to the pseudoinversion lemma in terms of an iterative computation of the Kalman gain from rank deficient autocorrelation matrices. The use of the Moore-Penrose pseudoinverse can be interpreted as inherent regularization. This can be illustrated by the relation of the singular value decomposition to the Moore-Penrose pseudoinverse and the truncation of the singular values.

In [8] a rigorous mathematical derivation of the pseudoinversion lemma can be found.

For clarity we introduce some predefinitions for the computation of the pseudo-inverse

$$\underline{\mathbf{f}}^{(\nu)} := \underline{\mathbf{X}}^{(\nu)}(m)\underline{\mathbf{S}}^{(\nu)\dagger}_{\mathbf{xx}}(m-1), \tag{27}$$

$$\underline{\mathbf{v}}^{(\nu)} := \underline{\mathbf{X}}^{(\nu)}(m)\left(\mathbf{I} - \underline{\mathbf{S}}^{(\nu)}_{\mathbf{xx}}(m-1)\underline{\mathbf{S}}^{(\nu)\dagger}_{\mathbf{xx}}(m-1)\right), \tag{28}$$

$$\beta^{(\nu)} := 1 + \underline{\mathbf{X}}^{(\nu)}(m)\underline{\mathbf{S}}^{(\nu)\dagger}_{\mathbf{xx}}(m-1)\underline{\mathbf{X}}^{(\nu)H}(m), \tag{29}$$

$$\underline{\mathbf{X}}^{(\nu)\dagger}(m) := \frac{\underline{\mathbf{X}}^{(\nu)H}(m)}{\left\|\underline{\mathbf{X}}^{(\nu)}(m)\right\|}. \tag{30}$$

In our case the matrix to be pseudo-inverted is a symmetric positive semi-definite matrix. Therefore, a proper iterative computation of the pseudoinverse of the autocorrelation matrix differentiates the two following cases depending on $\underline{\mathbf{v}}^{(\nu)}$ which represents the projection of $\underline{\mathbf{X}}^{(\nu)}(m)$ onto the kernel of $\underline{\mathbf{S}}^{(\nu)}_{\mathbf{xx}}(m-1)$

- the new data vector is not in the column range of the autocorrelation matrix i.e.,

$$\underline{\mathbf{X}}^{(\nu)}(m) \notin \mathcal{R}\left(\underline{\mathbf{S}}^{(\nu)}_{\mathbf{xx}}(m-1)\right) \Leftrightarrow \underline{\mathbf{v}} \neq \mathbf{0}. \tag{31}$$

  Here the pseudo-inverse can be computed with the formula [8]

$$\underline{\mathbf{S}}^{(\nu)\dagger}_{\mathbf{xx}}(m) = \underline{\mathbf{S}}^{(\nu)\dagger}_{\mathbf{xx}}(m-1) - 2\cdot\underline{\mathbf{v}}^{(\nu)\dagger}\underline{\mathbf{f}}^{(\nu)} + \beta^{(\nu)}\underline{\mathbf{v}}^{(\nu)\dagger}\underline{\mathbf{v}}^{(\nu)H\dagger}, \tag{32}$$

- the update data vector is in the column range of the autocorrelation matrix i.e.,

$$\underline{\mathbf{X}}^{(\nu)}(m) \in \mathcal{R}\left(\underline{\mathbf{S}}^{(\nu)}_{\mathbf{xx}}(m-1)\right) \Leftrightarrow \underline{\mathbf{v}} = \mathbf{0}. \tag{33}$$

  In this case the iterative computation of the pseudoinverse is equivalent to the inversion lemma namely,

$$\underline{\mathbf{S}}^{(\nu)\dagger}_{\mathbf{xx}}(m) = \underline{\mathbf{S}}^{(\nu)\dagger}_{\mathbf{xx}}(m-1) - \frac{1}{\beta^{(\nu)}}\underline{\mathbf{f}}^{(\nu)H}\underline{\mathbf{f}}^{(\nu)}. \tag{34}$$

  Unfortunately, the decision with a hard threshold could fail to detect real subspace changes. Therefore, a regularized inversion in the spatio-temporal transform domain as in Eq. (17) when $\left\|\underline{\mathbf{v}}^{(\nu)}\right\|$ is greater than a predefined threshold is recommended.

VI. Complexity considerations

The inversion of a diagonal matrix has a complexity in $O(L \cdot P)$, where P is the number of loudspeakers. Therefore, the computational costs of the first proposed approach in this paper are dominated by the complexity of the eigenvalue decomposition of the modified matrix. This has a computational cost of $O(L \cdot P^3)$. However, if the signal subspace is of the rank k the modified eigenproblem can be deflated and it can be shown, that the complexity of the update process will be in $O(L \cdot k \cdot P^2)$. Moreover, since this approach provides the eigenvectors of the signal correlation matrix, it allows performing the adaptive filtering in the signal subspace, which is often smaller than the system space.

The second approach performs a recursive update to the Moore-Penrose pseudoinverse similarly to the matrix inversion lemma, but in addition, it requires the computation of the correlation matrix as an auxiliary operation. However, no explicit inversion is required and, hence, the resulting complexity of the inversion is reduced to $O(L \cdot P^2)$.

VII. Experiments

To illustrate the properties of the proposed Kalman gain computation approaches, a multichannel AEC application scenario will be considered. The simulation setup consists of a circular array with 56 omni-directional loudspeakers and a radius of 1.5m and a rigid spherical array with 64 microphones and a radius of 0.075m, see Fig. 3. Both arrays are located in a room acoustically modeled by the image source method with an acoustic reflection factor at the walls of $\rho = 0.9$ and the dimension $[6\text{m}\,6\text{m}\,3\text{m}]$.

Rendering of the moving point source, in terms of simulating the far end room, is done by wave field synthesis (WFS, impulse responses are nearly Dirac impulses with different, suitably chosen delays and amplitudes). The point source is emitting white noise. Note that in these simulations we did not apply any pre-processing.

The performance of the algorithms is evaluated by means of the echo return loss enhancement (ERLE), see Fig. 4.

- The dotted green curve depicts the performance of the transform-domain adaptive filtering with recursive computation of the spatial transformation domain.
- The red curve is produced by reinitializing The estimation of the autocorrelation matrix when subspace changes are detected. It shows that the algorithms converges in this case to a local minimum due to the non-uniqueness problem.
- The blue curve shows the performance of the echo cancellation when the Kalman gain is computed by applying the pseudo-inversion lemma with regularization in the spatio-temporal transform domain when subspace changes are detected.

VIII. Conclusion

In this paper we presented two approaches for an efficient computation of the Kalman gain for multichannel adaptive filtering. The approach deals with the ill-conditioning of the spatial autocorrelation

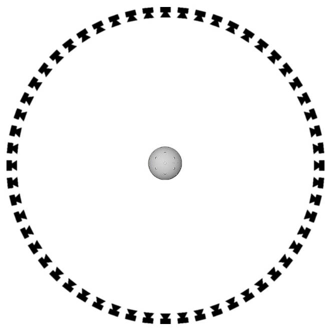

Fig. 3. Simulated echo cancellation scenario.

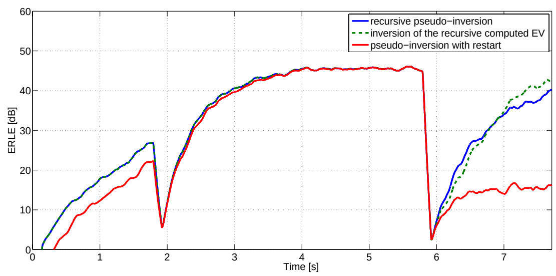

Fig. 4. Echo return loss enhancement.

matrix in multichannel systems and the computational complexity of multichannel adaptive filtering algorithms. The first approach bases on the rank-one modification of the eigenvalue problem. The second approach generalize the inversion lemma to ill-conditioned matrices by using a pseudoinversion lemma. The simulations have proved that the presented approaches offer the ability for rapid tracking of the signal subspace changes and high complexity reduction.

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