

AN OUTLIER-ROBUST EXTENDED MULTIDELAY FILTER WITH APPLICATION TO ACOUSTIC ECHO CANCELLATION

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ABSTRACT

We propose a novel class of efficient and robust adaptive algorithms in the frequency domain that is tailored to very long adaptive filters and highly autocorrelated input signals as they arise, e.g., in echo cancellation for high-quality full-duplex audio applications. The approach exhibits fast convergence, good tracking capabilities of the signal statistics, and very low delay. Moreover, the low order of computational complexity of the conventional frequency-domain adaptive algorithms can be maintained thanks to efficient realizations. The algorithm allows a tradeoff between the well-known multidelay filter (MDF) and the recursive least-squares (RLS) algorithm. It is also well suited for an efficient generalization to the multichannel case. Moreover, as the robustness issue during double talk is particularly crucial for fast-converging algorithms, we apply the concept of *robust statistics* into our extended frequency-domain approach. Due to the robust generalization of the cost function leading to a so-called M-estimator, the algorithms become inherently less sensitive to outliers, i.e., short bursts that may be caused by potential double-talk detection failures.

1. INTRODUCTION

Many signal processing applications require adaptive filters with very long impulse responses. In acoustic echo cancellation (AEC) as shown in Fig. 1, for example, thousands of FIR filter coefficients may be required to sufficiently model the echo path. Moreover, the input data are often very highly correlated which causes slow convergence of most algorithms [1]. The requirements are particularly demanding for high-quality and/or multichannel audio reproduction.

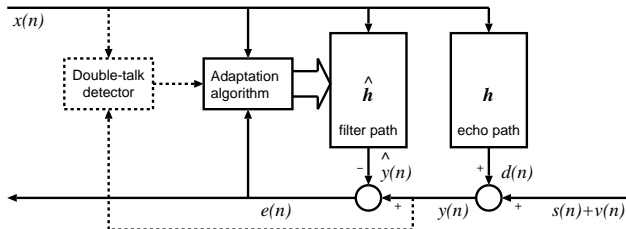


Fig. 1. Adaptive filter and double-talk detector in the AEC application.

An attractive solution to these problems is to use frequency-domain adaptive filters since, on the one hand, the computational

complexity can be greatly reduced by exploiting the Fast Fourier Transform (FFT). On the other hand, the Discrete Fourier Transform (DFT) approximately decorrelates the input signals, which leads to very favorable convergence properties of the adaptive algorithms. Frequency-domain methods rely on block-processing. In early approaches, the block length, i.e., the number of new samples used for each update, was set to the number of filter taps. The associated processing delay, equal to the block length, and the resulting difficulty to follow time-varying statistics of non-stationary signals, are often considered to be a major handicap. Therefore, a more flexible structure was introduced, the multidelay filter (MDF) [2], where the filter length L is partitioned into shorter length- N sub-filters. While the processing delay can be significantly reduced with this structure, the major disadvantage of choosing a block length N that is much shorter than the filter length L is that the convergence speed is often severely degraded for highly correlated signals since the correlations between these shorter blocks are not taken into account.

In this paper, we study an extended MDF (EMDF) to solve this problem. The EMDF algorithm in its baseline version [3] follows directly from a generic partitioned frequency-domain adaptive algorithm which can be rigorously derived from an exponentially weighted least-squares criterion in the frequency-domain [4]. The generic frequency-domain framework has led to efficient implementations of multichannel acoustic echo cancellation systems by inherently taking all inter-channel correlations into account. In a similar way, the EMDF algorithm contains all inter-partition correlations.

In addition to an improved echo cancellation performance, we also consider here the robust operation during double talk, i.e., during speaker activity in the receiving room. The robustness issue during double talk is particularly crucial for fast-converging algorithms as a failure of double-talk detection (Fig. 1) may then in turn cause fast divergence. Therefore, we apply the concept of *robust statistics* [5] to the extended frequency-domain approach. Due to the generalization of the cost function to a so-called M-estimator, the algorithms become inherently less sensitive to outliers, i.e., short bursts that may be caused by double-talk failures at the beginning or the end of utterances. Robust statistics has already been shown to be a very powerful tool to circumvent this problem in the echo cancellation application [6, 7, 8]. In this contribution, we show how this concept can be included in the generic partitioned frequency-domain algorithm for obtaining a robust EMDF realization. To keep the formal presentation short and accessi-

ble, we concentrate on the single-channel EMDF algorithm in this paper; the generalization to the multichannel version is obtained analogously as in [4]. In contrast to inter-channel correlations, the inter-partition correlations in the EMDF result from a shift-structure of the data. This structure can be exploited to derive fast implementations. Using a fast implementation of the EMDF algorithm (FEMDF) the computational complexity can be kept on the same order as that of the classical MDF.

2. ROBUST GENERIC PARTITIONED FREQUENCY-DOMAIN ADAPTIVE FILTERING

Here, we present a generic frequency-domain algorithm in its robust partitioned and constrained single-channel version providing the basis for the robust EMDF algorithm introduced in Sect. 3.

2.1. Definitions and Notation

In this paper, we follow the same notation as in [4], where a detailed derivation and an analysis of the non-robust generic algorithm can be found.

From Fig. 1, it can be seen that the error signal at time n between the output of the adaptive filter $\hat{y}(n)$ and the desired output signal $y(n)$ is given by

$$e(n) = y(n) - \sum_{\kappa=0}^{L-1} x(n - \kappa) \hat{h}_{\kappa}, \quad (1)$$

where \hat{h}_{κ} are the coefficients of the filter impulse response. By partitioning the impulse response \hat{h} of length L into K segments of length $N = L/K$ as in [2], (1) can be written as

$$\begin{aligned} e(n) &= y(n) - \sum_{k=0}^{K-1} \sum_{\kappa=0}^{N-1} x(n - Nk - \kappa) \hat{h}_{Nk+\kappa} \\ &= y(n) - \sum_{k=0}^{K-1} \mathbf{x}_k^T(n) \hat{\mathbf{h}}_k = y(n) - \mathbf{x}^T(n) \hat{\mathbf{h}}, \end{aligned} \quad (2)$$

where

$$\mathbf{x}_k(n) = [x(n - Nk), x(n - Nk - 1), \dots, x(n - Nk - N + 1)]^T, \quad (3)$$

$$\hat{\mathbf{h}}_k = [\hat{h}_{Nk}, \hat{h}_{Nk+1}, \dots, \hat{h}_{Nk+N-1}]^T, \quad (4)$$

$$\mathbf{x}(n) = [\mathbf{x}_0^T(n), \mathbf{x}_1^T(n), \dots, \mathbf{x}_{K-1}^T(n)]^T. \quad (5)$$

Superscript T denotes transposition of a vector or a matrix. The length- N vectors $\hat{\mathbf{h}}_k$, $k = 0, \dots, K - 1$ represent *sub-filters* of the partitioned tap-weight vector

$$\hat{\mathbf{h}} = [\hat{\mathbf{h}}_0, \dots, \hat{\mathbf{h}}_{K-1}]^T. \quad (6)$$

We now define the block error signal of length N . Based on (2) we write

$$\mathbf{e}(m) = \mathbf{y}(m) - \sum_{k=0}^{K-1} \mathbf{U}_k^T(m) \hat{\mathbf{h}}_k, \quad (7)$$

where m is the block time index, and

$$\mathbf{e}(m) = [e(mN), \dots, e(mN + N - 1)]^T, \quad (8)$$

$$\mathbf{y}(m) = [y(mN), \dots, y(mN + N - 1)]^T, \quad (9)$$

$$\mathbf{U}_k(m) = [\mathbf{x}_k(mN), \dots, \mathbf{x}_k(mN + N - 1)]. \quad (10)$$

To derive the frequency-domain algorithm, the block error signal (7) is transformed by a DFT matrix to its frequency-domain counterpart. The matrices $\mathbf{U}_k(m)$, $k = 0, \dots, K - 1$ are Toeplitz matrices of size $(N \times N)$. Since a Toeplitz matrix $\mathbf{U}_k(m)$ can be transformed, by doubling its size, to a circulant matrix of size $(2N \times 2N)$, and a circulant matrix can be diagonalized using the $(2N \times 2N)$ -DFT matrix \mathbf{F}_{2N} with elements $e^{-j2\pi\nu n/(2N)}$ ($\nu, n = 0, \dots, 2N - 1$), we have

$$\mathbf{U}_k^T(m) = \underbrace{[\mathbf{0}_{N \times N}, \mathbf{I}_{N \times N}] \mathbf{F}_{2N}^{-1} \mathbf{X}_k(m) \mathbf{F}_{2N}}_{:= \mathbf{W}_{N \times 2N}^{01}} \underbrace{[\mathbf{I}_{N \times N}, \mathbf{0}_{N \times N}]^T}_{:= \mathbf{W}_{2N \times N}^{10}}$$

with the diagonal matrices

$$\mathbf{X}_k(m) = \text{diag}\{\mathbf{F}_{2N}[x(mN - Nk - N), \dots, x(mN - Nk + N - 1)]^T\}. \quad (11)$$

This finally leads to the following block error signal

$$\mathbf{e}(m) = \mathbf{y}(m) - \mathbf{W}_{N \times 2N}^{01} \mathbf{F}_{2N}^{-1} \mathbf{X}(m) \mathbf{G}_{2L \times L}^{10} \hat{\mathbf{h}}, \quad (12)$$

where

$$\mathbf{X}(m) = [\mathbf{X}_0(m), \mathbf{X}_1(m), \dots, \mathbf{X}_{K-1}(m)], \quad (13)$$

$$\mathbf{G}_{2L \times L}^{10} = \text{diag}\{\mathbf{G}_{2N \times N}^{10}, \dots, \mathbf{G}_{2N \times N}^{10}\}, \quad (14)$$

$$\mathbf{G}_{2N \times N}^{10} = \mathbf{F}_{2N} \mathbf{W}_{2N \times N}^{10} \mathbf{F}_N^{-1}, \quad (15)$$

$$\hat{\mathbf{h}} = [\hat{\mathbf{h}}_0^T, \dots, \hat{\mathbf{h}}_{K-1}^T]^T, \quad (16)$$

$$\hat{\mathbf{h}}_k = \mathbf{F}_N \hat{\mathbf{h}}_k. \quad (17)$$

2.2. Adaptation Algorithm

As in derivations of time-domain adaptive algorithms, we now form a criterion that is minimized with respect to its filter coefficients. Modeling the noise with a probability density function (PDF) with a tail that is heavier than the Gaussian PDF gives us a non-quadratic function to minimize, which results in an outlier-robust algorithm [5]. Following [5] we choose to work with the following criterion:

$$\tilde{J}(\hat{\mathbf{h}}) = \sum_{n=mN}^{mN+N-1} \rho \left[\frac{|e(n)|}{s} \right], \quad (18)$$

where $\rho[\cdot]$ is a convex function and s is a real positive scale factor for block m , as discussed in [7]. The resulting algorithm inherits its robust properties as long as the nonlinear function $\rho[\cdot]$ has a bounded derivative [5]. The so-called Huber estimator is given by the following choice of $\rho[\cdot]$ [5]:

$$\rho(|z|) = \begin{cases} \frac{|z|^2}{2}, & \text{for } |z| \leq k_0, \\ k_0|z| - \frac{k_0^2}{2}, & \text{for } |z| \geq k_0. \end{cases} \quad (19)$$

where k_0 is a constant controlling the robustness of the algorithm. Adaptive Newton-type algorithms [9] minimize the criterion (18) by using a recursion of the form

$$\hat{\mathbf{h}}(m) = \hat{\mathbf{h}}(m-1) - \mu' \mathbf{S}_{\psi'}^{-1} \nabla \tilde{J} \hat{\mathbf{h}}(m-1), \quad (20)$$

where $\nabla \tilde{J}(\hat{\mathbf{h}}) = \partial / \partial \hat{\mathbf{h}} \tilde{J}$ is the gradient of the optimization criterion w.r.t. $\hat{\mathbf{h}}$, $\mathbf{S}_{\psi'}$ is an approximation of the expected value of

the Hessian $\nabla^2 \tilde{J} = \partial / \partial \hat{\mathbf{h}}^* (\nabla \tilde{J})^H$ which will be specified below in more detail, and μ' is the relaxation parameter.

To proceed, we need to calculate the gradient and Hessian of (18). We write one complex conjugated element of the error vector $\mathbf{e}(m)$ as

$$\begin{aligned} e^*(n) &= \mathbf{e}^H(m) \mathbf{1}_{n-mN} \\ &= \left[\mathbf{y}^H(m) - \hat{\mathbf{h}}^H (\mathbf{G}_{2L \times L}^{10})^H \mathbf{X}^H(m) \mathbf{F}_{2N}^{-H} \mathbf{W}_{2N \times N}^{01} \right] \\ &\quad \cdot \mathbf{1}_{n-mN}, \end{aligned} \quad (21)$$

where $n = mN, \dots, mN + N - 1$, and $\mathbf{1}_i$ is a length- N vector containing a 1 in position i and zeros in all other positions. Then, the gradient is found using the chain rule:

$$\begin{aligned} \nabla \tilde{J} &= \frac{\partial}{\partial \hat{\mathbf{h}}^*} \tilde{J} = \sum_{n=mN}^{mN+N-1} \frac{\partial}{\partial \hat{\mathbf{h}}^*} \rho \left[\frac{|e(n)|}{s} \right] \\ &= \sum_{n=mN}^{mN+N-1} \frac{\partial}{\partial \hat{\mathbf{h}}^*} [e^*(n)] \rho' \left[\frac{|e(n)|}{s} \right] \frac{\text{sign}[e(n)]}{s} \end{aligned}$$

Using (21), it follows

$$\begin{aligned} \nabla \tilde{J} &= -\frac{1}{2Ns} (\mathbf{G}_{2L \times L}^{10})^H \mathbf{X}^H(m) \mathbf{F}_{2N} \mathbf{W}_{2N \times N}^{01} \underline{\boldsymbol{\psi}}[\mathbf{e}(m)] \\ &= -\frac{1}{2Ns} (\mathbf{G}_{2L \times L}^{10})^H \mathbf{X}^H(m) \underline{\boldsymbol{\psi}}[\mathbf{e}(m)], \end{aligned} \quad (22)$$

where

$$\underline{\boldsymbol{\psi}}[\mathbf{e}(m)] = \begin{bmatrix} \psi \left[\frac{|e(mN)|}{s} \right] \text{sign}[e(mN)] \\ \vdots \\ \psi \left[\frac{|e(mN+N-1)|}{s} \right] \text{sign}[e(mN+N-1)] \end{bmatrix},$$

$$\psi(|z|) = \rho'(|z|) = \min\{|z|, k_0\},$$

$$\underline{\boldsymbol{\psi}}[\mathbf{e}(m)] = \mathbf{F}_{2N} \begin{bmatrix} \mathbf{0}_{N \times N} \\ \underline{\boldsymbol{\psi}}[\mathbf{e}(m)] \end{bmatrix}.$$

Combining (20) and (22), we find the robust frequency-domain update

$$\hat{\mathbf{h}}(m) = \hat{\mathbf{h}}(m-1) + \frac{\mu'}{2Ns} \mathbf{S}_{\psi'}^{-1}(m) (\mathbf{G}_{2L \times L}^{10})^H \mathbf{X}^H(m) \underline{\boldsymbol{\psi}}[\mathbf{e}(m)]. \quad (23)$$

We now derive the Hessian and estimate its expected value $\mathbf{S}_{\psi'}$. The Hessian is expressed as

$$\begin{aligned} \nabla^2 \tilde{J} &= \frac{\partial}{\partial \hat{\mathbf{h}}^*} (\nabla \tilde{J})^H \\ &= -\frac{1}{s} \frac{\partial}{\partial \hat{\mathbf{h}}^*} \boldsymbol{\psi}^H[\mathbf{e}(m)] \mathbf{W}_{N \times 2N}^{01} \mathbf{F}_{2N}^{-1} \mathbf{X}(m) \mathbf{G}_{2L \times L}^{10}. \end{aligned}$$

By applying again the chain rule, followed by recursive averaging using a forgetting factor λ ($0 < \lambda < 1$), we finally obtain the estimate

$$\begin{aligned} \mathbf{S}_{\psi'}(m) &= \frac{(1-\lambda)}{s^2} \sum_{i=0}^m \lambda^{m-i} (\mathbf{G}_{2L \times L}^{10})^H \mathbf{X}^H(i) \tilde{\mathbf{G}}_{2N \times 2N}(i) \\ &\quad \cdot \mathbf{X}(i) \mathbf{G}_{2L \times L}^{10}, \end{aligned} \quad (24)$$

where

$$\tilde{\mathbf{G}}_{2N \times 2N}(m) = \mathbf{F}_{2N}^{-H} \mathbf{W}_{2N \times N}^{01} \boldsymbol{\Psi}'[\mathbf{e}(m)] \mathbf{W}_{N \times 2N}^{01} \mathbf{F}_{2N}^{-1}, \quad (25)$$

$$\boldsymbol{\Psi}'[\mathbf{e}(m)] = \text{diag} \left\{ \psi' \left[\frac{|e(mN)|}{s} \right] \dots \psi' \left[\frac{|e(mN+N-1)|}{s} \right] \right\},$$

and ψ' is the derivative of ψ , i.e., $\psi'(|z|)$ is 1 for $|z| \leq k_0$ and 0 else.

Eqs. (24), (12), and (23) form the main equations of the generic adaptive algorithm. In the same way as shown in [4], these equations can be reformulated in a practically more useful form:

$$\begin{aligned} \mathbf{S}_{xx}(m) &= \lambda \mathbf{S}_{xx}(m-1) \\ &\quad + \frac{(1-\lambda)}{s^2} \mathbf{X}^H(m) \tilde{\mathbf{G}}_{2N \times 2N}(m) \mathbf{X}(m), \end{aligned} \quad (26)$$

$$\mathbf{K}(m) = \mathbf{S}_{xx}^{-1}(m) \mathbf{X}^H(m), \quad (27)$$

$$\mathbf{e}(m) = \mathbf{y}(m) - \mathbf{W}_{N \times 2N}^{01} \mathbf{F}_{2N}^{-1} \mathbf{X}(m) \hat{\mathbf{h}}_{2L}(m-1), \quad (28)$$

$$\hat{\mathbf{h}}_{2L}(m) = \hat{\mathbf{h}}_{2L}(m-1) + \frac{\mu'}{2Ns} \mathbf{G}_{2L \times 2L}^{10} \mathbf{K}(m) \underline{\boldsymbol{\psi}}[\mathbf{e}(m)]. \quad (29)$$

Due to the formal similarity of Eqs. (26)-(29) to the RLS algorithm [1] in the time domain, we call the matrix $\mathbf{K}(m)$ the frequency-domain Kalman gain. The Kalman gain plays a key role in the following sections.

3. ROBUST EXTENDED MULTIDELAY FILTER (EMDF)

The Algorithm (26)-(29) is strictly equivalent to the (robust) RLS algorithm in the time domain for a block length $N = 1$. Unfortunately, the matrix $\mathbf{S}_{xx}(m)$ in (26) is not diagonal, so the above generic algorithm still has a high computational complexity due to the matrix inversion in (27).

To simplify this algorithm we would like to use the same approximation as for the non-robust frequency-domain method in [4]. We start by approximating $\boldsymbol{\Psi}'[\mathbf{e}(m)]$ as

$$\boldsymbol{\Psi}'[\mathbf{e}(m)] = \psi'_{\min}(m) \mathbf{I}_{N \times N}, \quad (30)$$

where $\psi'_{\min}(m)$ should be bounded by $\mu = \mu'/(1-\lambda)$ to prevent instability ($\psi' \in \{0, 1\}$) of the update (29),

$$\psi'_{\min}(m) = \max \left[\mu, \min_{0 \leq n \leq N-1} \left\{ \psi' \left[\frac{|e(mN+n)|}{s} \right] \right\} \right].$$

Since $\mathbf{F}_{2N}^{-H} \mathbf{W}_{2N \times 2N}^{01} \mathbf{F}_{2N}^{-1} = \mathbf{G}_{2N \times 2N}^{01}/(2N)$, (25) is now approximated as

$$\tilde{\mathbf{G}}_{2N \times 2N}(m) = \frac{\psi'_{\min}(m)}{2N} \mathbf{G}_{2N \times 2N}^{01}. \quad (31)$$

Furthermore, as shown in [4], matrix $\mathbf{G}_{2N \times 2N}^{01}$ can very well be approximated by $\mathbf{G}_{2N \times 2N}^{01} = \mathbf{I}_{2N \times 2N}/2$ in (26) for sufficiently large N . This approximation leads to a block-diagonal structure of matrix $\mathbf{S}_{xx}(m)$ with the diagonal sub-matrices ($i, j = 0, \dots, K-1$)

$$\mathbf{S}_{i,j}(m) = \lambda \mathbf{S}_{i,j}(m-1) + (1-\lambda) \mathbf{X}_i^*(m) \mathbf{X}_j(m). \quad (32)$$

Figure 2 illustrates the block structure for the example of 5 partitions. The classical multidelay filter (MDF) in its robust version is obtained by further approximating $\mathbf{S}_{xx}(m)$ by dropping the off-diagonal components, i.e., the inter-partition correlations (grey diagonals in Fig. 2). This leads to the low computational complexity per output sample, which is linear in K .

The extended multidelay filter (EMDF) takes the inter-partition correlations into account and thus provides a better approximation to the exact solution of the normal equation. However, a straightforward implementation leads to a computational complexity, which increases quadratically with the number K of partitions. Fast schemes, as discussed in the next section, provide a solution with a complexity that is comparable to that of the classical MDF.

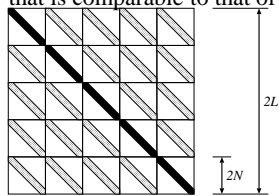


Fig. 2. Structure of matrix $S_{x,x}(m)$.

4. FAST IMPLEMENTATIONS OF THE ROBUST EMDF

To reduce the computational complexity of the robust EMDF algorithm, it is interesting that the data among the partitions are not independent. Due to the formal similarity of Eqs. (32), (27)-(29) with the RLS algorithm in the time domain [1, 10], corresponding *fast* implementations of the Kalman gain (Eqn. (27) which turns out to be the same for the robust and non-robust versions) can be expected. In [3] it is shown that all fast calculation schemes known for the RLS can actually be applied to the EMDF after a slight modification. The key for fast RLS realizations is the shift-structure of the input signal vector [1, 10]. In case of the (robust) EMDF there is a corresponding shift-structure among the partitions (in each frequency-bin). Using this approach the complexity increases only linearly (instead of quadratically as with the ordinary EMDF algorithm) with the number of partitions, and the overall complexity is on the same order as in the classical MDF. As an example, a fast EMDF algorithm based on the so-called fast transversal filter (FTF) structure [10] is given in [3].

5. EVALUATION FOR ACOUSTIC ECHO CANCELLATION

We demonstrate the performance of the algorithm by an example for acoustic echo cancellation. We apply the (single-channel) EMDF algorithm for (single-channel) AEC with $K = 50$ partitions, a block length (each partition) $N = 64$, and a high sampling rate of 48 kHz. As input signal, we chose classical music (*Air* by Bach). The signal sequence is highly auto-correlated (tonal sounds, which are known as worst case for the adaptation). In Fig. 3 we compare different algorithms without doubletalk. In this case, an echo-to-background noise ratio (EBR) of 45 dB on the microphone was chosen. The dashed and dash-dotted lines in Fig. 3 show the *echo return loss enhancement* $ERLE$ and the coefficient error norm achieved by the conventional MDF and the fast RLS, respectively, as the extreme cases. For the solid lines, the same data and the same parameters are used with the EMDF algorithm. It is important to note that the regularization is adjusted in each case. Several simulations have confirmed that the EMDF shows a significantly more stable behaviour than the classical MDF due to the more accurate approximation to the exact recursive solution of the normal equation while the complexity is kept low (MULs for Kalman gain per output sample: in our example FRLS 16000, MDF 100, FEMDF (FTF) 250). In Fig. 4, we compare the coefficient error norm of the robust and non-robust EMDF algorithms in the double-talk case with a coherence-based detector [11].

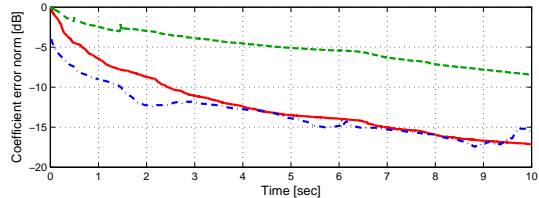
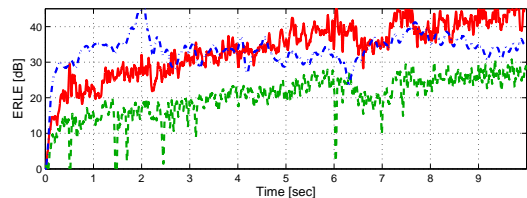


Fig. 3. Comparison between classical MDF (dashed lines), Fast RLS (dash-dot), and EMDF (solid lines).

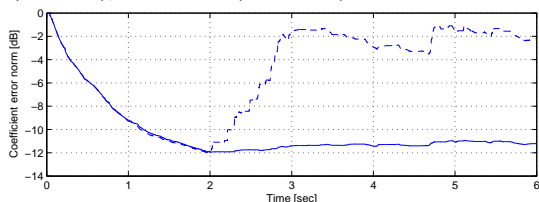


Fig. 4. Double-talk case: near-end speech after 2sec. Non-robust EMDF (dashed line) and robust EMDF (solid line).

6. REFERENCES

- [1] S. Haykin, *Adaptive Filter Theory*, 3rd ed., Prentice Hall Inc., Englewood Cliffs, NJ, 1996.
- [2] J.-S. Soo and K.K. Pang, "Multidelay block frequency domain adaptive filter," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. ASSP-38, pp. 373-376, Feb. 1990.
- [3] H. Buchner, J. Benesty, and W. Kellermann, "An extended multidelay filter: fast low-delay algorithms for very high-order adaptive systems," in *Proc. IEEE ICASSP*, April 2003.
- [4] H. Buchner, J. Benesty, and W. Kellermann, "Multichannel Frequency-Domain Adaptive Algorithms with Application to Acoustic Echo Cancellation," in J. Benesty and Y. Huang (eds.), *Adaptive signal processing: Application to real-world problems*, Springer-Verlag, Berlin/Heidelberg, Jan. 2003.
- [5] P.J. Huber, *Robust Statistics*, Wiley, New York, 1981.
- [6] T. Gänslér, "A double-talk resistant subband echo canceller," *Signal Processing*, vol. 65, pp. 89-101, Feb. 1998.
- [7] T. Gänslér, S.L. Gay, M.M. Sondhi, and J. Benesty, "Double-talk robust fast converging algorithms for network echo cancellation," *IEEE Trans. Speech Audio Processing*, vol. 8, pp. 656-663, Nov. 2000.
- [8] J. Benesty and T. Gänslér, "A robust fast recursive least-squares adaptive algorithm," in *Proc. IEEE ICASSP*, 2001.
- [9] S. M. Kay, *Fundamentals of Statistical Signal Processing: Estimation Theory*, Prentice Hall PTR, Upper Saddle River, NJ, 1993.
- [10] M.G. Bellanger, *Adaptive Digital Filters and Signal Analysis*, Marcel Dekker, 1987.
- [11] J. Benesty, D.R. Morgan, and J.H. Cho, "A new class of doubletalk detectors based on cross-correlation," *IEEE Trans. Speech Audio Processing*, vol. 8, pp. 168-172, Mar. 2000.